17-5 day 1 The Normal Distribution

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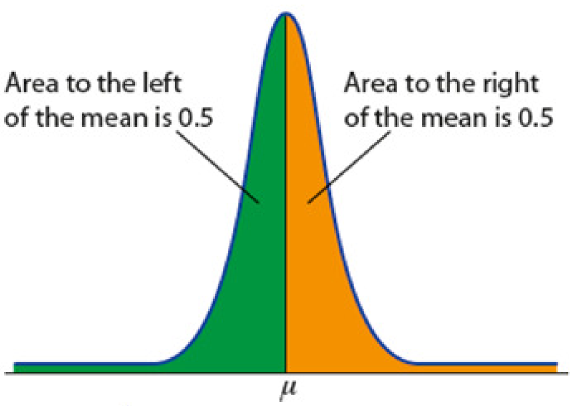
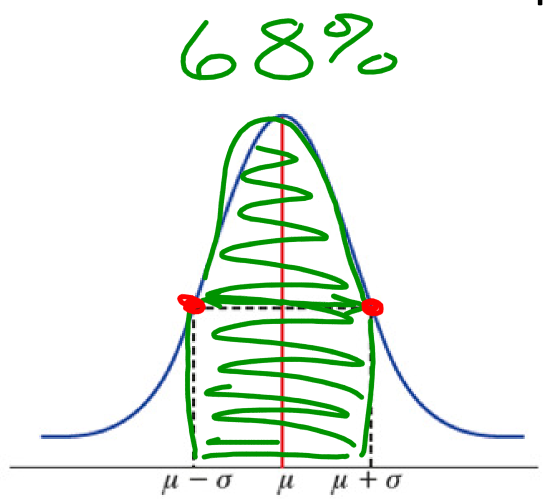
The normal distribution is a continuous pdf that is symmetrical about the mean and has a bell shaped curve.





Hence, the mean and median are the same.

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Probability Density Function of the Normal Distribution

The probability density function for a normally distributed random variable x is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where μ and σ^2 are any number such that

$-\infty < \mu < \infty$ and $0 \leq \sigma^2 < \infty$

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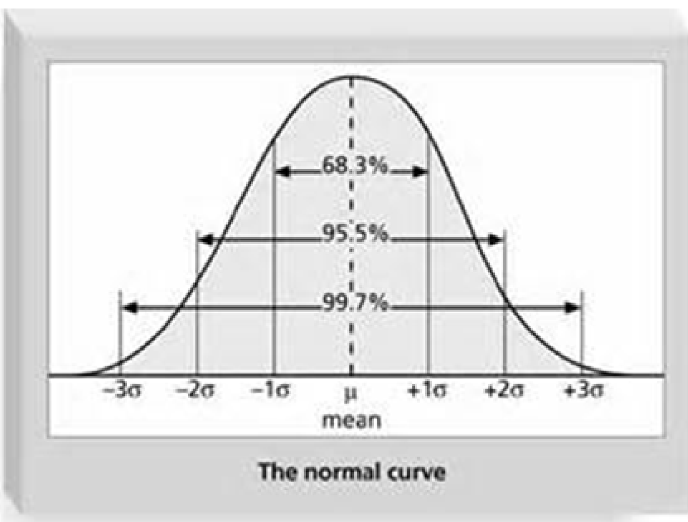
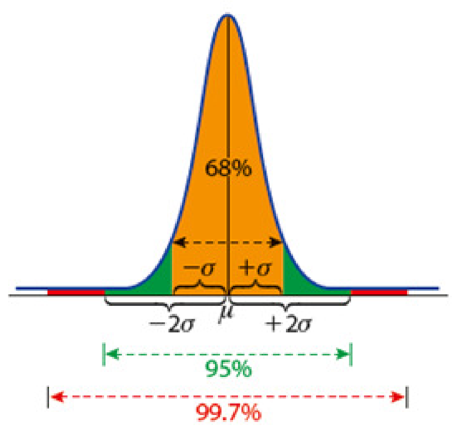
When a variable is normally distributed, we

$$x \sim N(\mu, \sigma^2)$$





When the variable is normally distributed, the mean is at the max, hence the derivative is zero.

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$f'(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\cdot-\left(\frac{x-\mu}{\sigma}\right)\cdot\frac{1}{\sigma}$$

$$f'(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}(x-\mu)e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$0=\frac{-1}{\sigma^3\sqrt{2\pi}}(x-\mu)e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$0=(x-\mu)e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$0=(x-\mu)\quad x=\mu$$

or

$$0=e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

no solution

Hence the mean is at

$$x=\mu$$

Now let's find where the inflection points are:

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$f'(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}(x-\mu)e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}\left[1\cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}+(x-\mu)e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\cdot-\left(\frac{x-\mu}{\sigma}\right)\cdot\frac{1}{\sigma}\right]$$

$$f''(x)=\frac{-1}{\sigma^3\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\left[1-\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

$$0=\frac{-1}{\sigma^3\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\left[1-\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

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Now let's find where the inflection points are:

$$0 = \frac{-1}{\sigma^3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\left[1-\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

$$0 = e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}\left[1-\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

$$0 = e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$ *No Solution*

or

$$0 = 1-\left(\frac{x-\mu}{\sigma}\right)^2$$

$$1 = \left(\frac{x-\mu}{\sigma}\right)^2$$

$$\sigma^2 = (x-\mu)^2$$

$$\pm\sigma = x-\mu$$

$$x = \mu \pm \sigma$$

Hence the inflection points are one standard deviation away from the mean.

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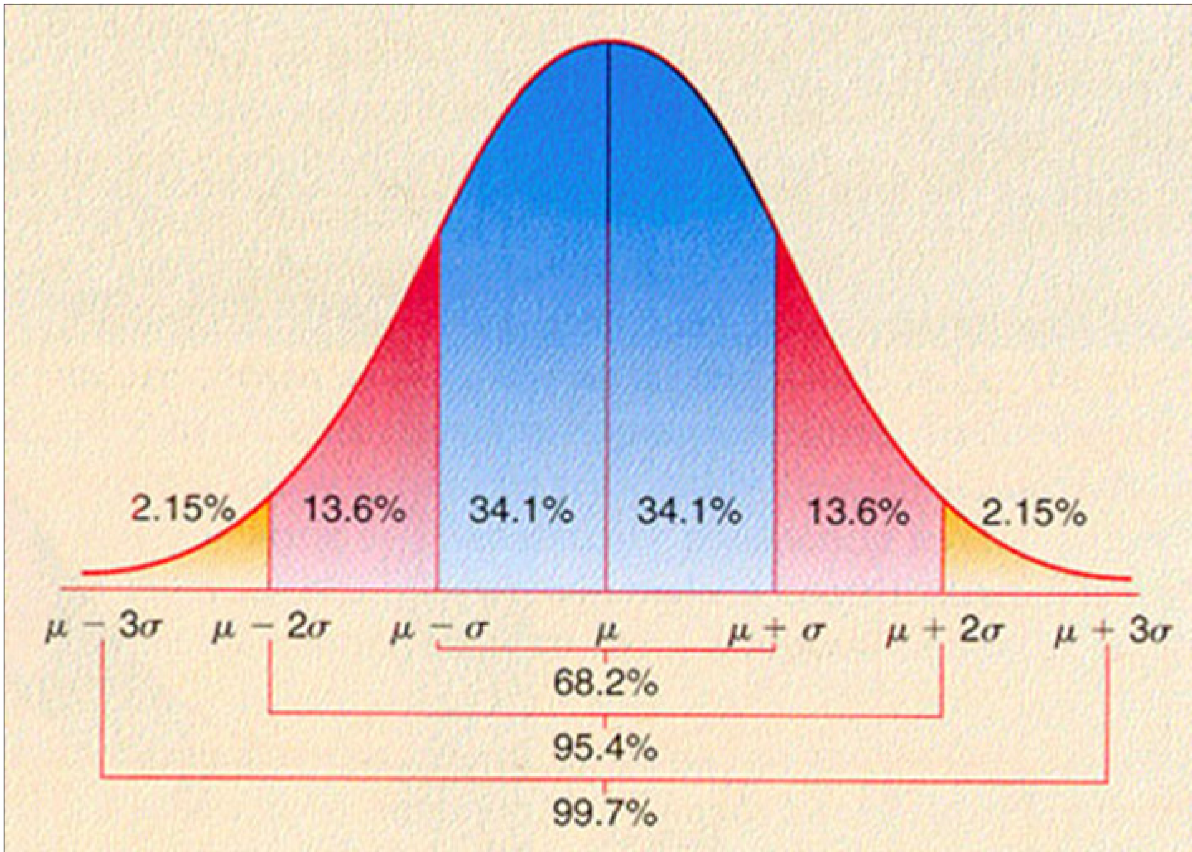

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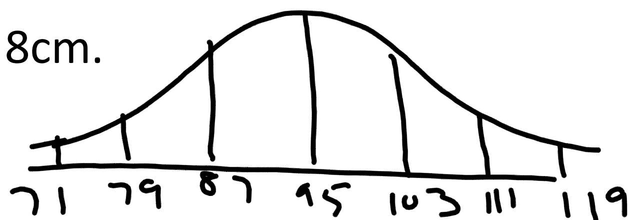
Ex1. The chest measurements of 18 year old male football players are normally distributed with a mean of 95 cm and a standard deviation of 8cm.



a.) Find the percentage of footballers with chest measurements between:

i. 87 cm and 103 cm 68.3%

ii. 103 cm and 111 cm 13.6%

iii. 103 cm and 119 cm 15.7%

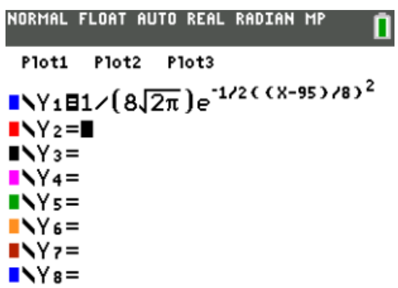
iv. Above 119 cm .15%

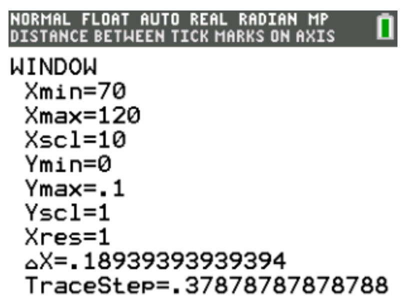
b.) Find the probability that a randomly chosen footballer has a chest measurement between 87 cm and 111 cm.

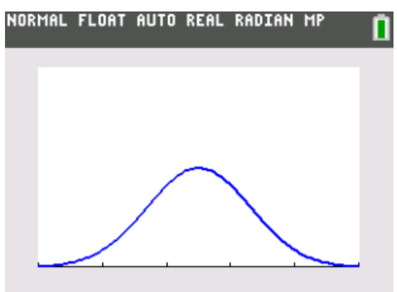
$\approx 81.9\%$

c.) Find the probability that a randomly chosen footballer has a chest measurement between 90 cm and 100cm.

$$pdf = \frac{1}{8\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-95}{8}\right)^2}$$

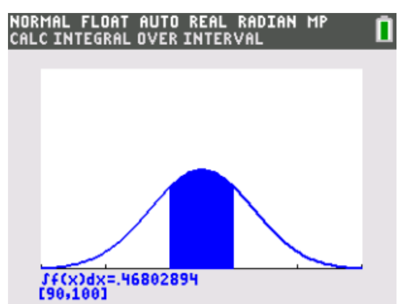


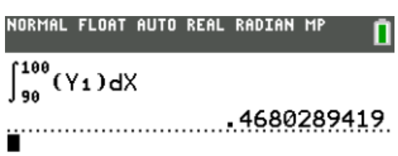




$$\int_{90}^{100} \frac{1}{8\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-95}{8}\right)^2} dx$$

$\approx \boxed{.468}$



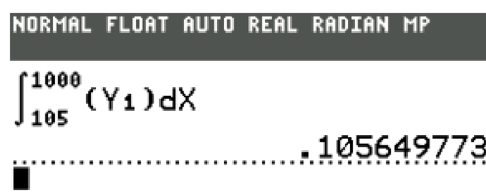


d.) Find the probability that a randomly chosen footballer has a chest measurement above 105 cm.

$$\int_{105}^{\infty} \frac{1}{8\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-95}{8}\right)^2}$$

$$1-\int_{0}^{105} \frac{1}{8\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-95}{8}\right)^2} dx$$

OR



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